JOURNAL OF MATERIALS SCIENCE 26 (1991) 1253–1258

The kinematics of an active zone during fatigue crack layer growth in polystyrene

J. BOTSIS, X. Q. ZHANG
Department of Civil Engineering, Mechanics and Metallurgy, University of Illinois at Chicago, Box 4348, Chicago, Illinois 60680, USA

An investigation of the kinematics of an active zone (or process zone) evolution in polystyrene during fatigue fracture is reported. Experiments were conducted on tension–tension single-edge-notched specimens of 0.25 mm thickness. Craze characterization was carried out on thinned sections of the active zone at six consecutive configurations. Analysis consisted of quantitative comparison of ratios of the inertia moments of the active zone at consecutive configurations. The results indicate that for the particular loading history considered, damage evolution can be approximated by a linear transformation of the space variables. The fracture process can be described by the translation and deformation of the active zone. Consequently, the corresponding energy release rates can be expressed by the J_1, M and N_{ij} integrals. The results of this analysis agree with the kinematics proposed by the crack layer model.

1. Introduction and background

To model the slow propagation of cracks in different materials researchers have considered formulations based on concepts of fracture mechanics. As a result, several fatigue crack propagation (FCP) models have been proposed which relate crack speed to some function of the stress intensity factor, K_1, for linear elastic fracture mechanics, or the energy release rate, J_1, for non-linear fracture mechanics [1]. Although, these models may well describe FCP data under specific loading histories, their applicability to a wide range of propagation rates is limited. For instance, experimental work in different materials has shown that crack deceleration occurs with increasing K_1 or J_1 [2–5]. This type of behaviour cannot be described by models based on conventional fracture mechanics parameters for they all predict a monotonic increase of crack speed with the increase in either K_1 or J_1.

In recent years a significant amount of experimental work has been accumulated which demonstrates that a zone of severely damaged material, adjacent to the crack tip, precedes stable crack growth [4, 6–16]. This zone of damage is usually called a plastic zone [1], process zone [17, 18], active zone [19, 20], etc. Although macroscopically the fracture process may be deterministic, the statistics of microdefect distribution in the vicinity of a crack tip controls crack propagation. Moreover, the nature of damage encountered in most cases is not necessarily of the type described by models of plasticity theory. Instead, the elements of damage are specific to the material [4, 6–16].

An approach based on the observation that the evolution of a damage zone accompanying a propagating crack displays similar features in spite of the differences in material structure and loading history, has been proposed [19, 20]. The similarity in evolution of a process zone permits the use of integral damage parameters to characterize a crack and its associated damage zone as one entity, a crack layer (CL), and introduce their driving forces within the framework of irreversible thermodynamics [19–21]. Therefore, instead of attempting to characterize a fracture process in terms of the kinetics of individual damage elements it is suggested that the damage zone be considered as one single entity and a model of the fracture process is made in terms of its constitutive response. Accordingly the characterization of the kinematic parameters of a process zone is essential.

A schematic presentation of a CL is shown in Fig. 1. The active zone is the domain of a CL where the rate of damage parameter is non-zero; $\dot{\rho}(\boldsymbol{x}) > 0$. The inert zone is the complementary part of the CL with $\dot{\rho}(\boldsymbol{x}) = 0$ and $\rho(\boldsymbol{x}) > 0$, where $\boldsymbol{x}$ is a position vector. The active zone can be roughly characterized by its width, w, and length, l_a. According to a self similarity hypothesis of damage evolution [19, 20], the rate of damage density, $\rho(\boldsymbol{x})$, at a point $\boldsymbol{x}$ within the active zone in a coordinate system attached to its gravity centre is [22]

$$\dot{\rho} = -V_i(\boldsymbol{x})\partial_i\rho(\boldsymbol{x})$$

where $\boldsymbol{x}$ is the position vector, $V_i(\boldsymbol{x})$ is the velocity of the corresponding point within the active zone and ∂ stands for partial derivative. The velocity $V_i(\boldsymbol{x})$ is approximated by the first two terms of a Taylor series around the gravity centre

$$V_i(\boldsymbol{x}) = V_i(\boldsymbol{0}) + V_{i,j}(\boldsymbol{0})x_j$$

The first term represents the rigid translation of the zone and the second the rotation (anti-symmetric part

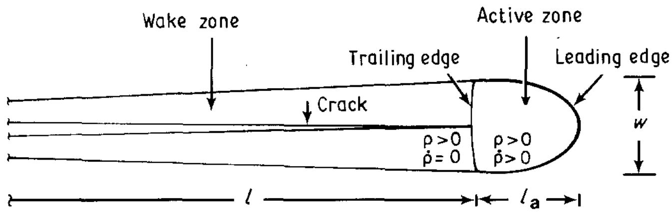

Figure 1 A schematic drawing of a crack layer.

of $V_{i,j}$) and deformation (symmetric part of $V_{i,j}$) which in turn can be decomposed into isotropic expansion and homogeneous distortion of the zone. This particular form of self-similarity of damage evolution allows description of a fracture process in terms of translation, rotation, isotropic expansion and homogeneous distortion of the zone. Moreover, it permits expression of the corresponding energy release rates in the form of J_i, L, M and N_{ij} integrals [19, 20]. Thus four kinematic parameters are introduced to describe a fracture process. Analysis of a process zone kinematics and identification of the corresponding energy release rates have also been reported in [23]. However, Aoki *et al.* [23] treat a process zone as a homogeneous solid capable of undergoing rigid body motion and homogeneous deformation. The results are similar to the CL approach described elsewhere [21, 24].

Although linear and non-linear fracture mechanics appear to be particular cases of the approaches mentioned above, there are no solid experimental results to justify the representation of an active zone by rigid body motion and homogeneous deformation. In this paper we investigate the kinematics of damage during rectilinear quasi-static fracture and examine the self-similarity hypothesis adopted by Chudnorsky [19, 20]. This is achieved by investigating damage distribution within an active zone at different instances during its evolution. The material employed in these studies is polystyrene (PS), an amorphous polymer. This material, besides being transparent, preserves damage patterns induced during fracture for relatively long periods of time, thus facilitating damage analysis. The experimental results presented here consist of craze density measurements within an active zone at different configurations. The analytical study is aimed at deducing the transformation of damage distribution at different instances of active zone evolution.

2. Experimental procedure

Single-edge-notched specimens, 0.25 mm thick, 80 mm gauge length and 20 mm wide, of plane isotropic PS were used in this investigation. The details of specimen preparation can be found in [6]. Tension–tension fatigue experiments were conducted on a 11.10 kN capacity servohydraulic two-actuator Instron testing system in a laboratory atmosphere using a sinusoidal wave form loading function at a frequency of 1.11 Hz, $\sigma_{max} = 15.2$ MPa and a load ratio of 0.28. An experiment is conducted initially, in order to record the crack growth kinetics and obtain the interval of stable CL propagation. Utilizing a method to identify the critical crack length, l_c, which is described in [6], the resulting value of l_c is 7.5 mm.

Beyond a crack length of about 6 mm the crack grows relatively fast and with large jumps. This makes a controlled interruption of the experiment difficult. Thus we analyse damage distribution within active zones grown to the crack length of 5.5 mm.

The kinematics of the active zone are investigated by comparing crazing distribution at six different configurations. Accordingly, six experiments are performed and interrupted when the respective crack lengths reach the values of 2.2, 3.6, 3.8, 4.5, 4.8 and 5.5 mm.

Crack propagation and the evolution of the surrounding damage are observed by means of a Questar long-range microscope. The fracture process is recorded using a Hamamatsu video system (recording speed = 30 frames/sec) which is attached to the microscope. Craze distribution is evaluated from optical micrographs of sectioned specimens (approximately 10 to 20 μm thick) which are prepared by standard metallographic and polishing procedures [25].

3. Results and discussion

In a study reported by Botsis [26], the kinematic parameters are deduced from the evolution of the characteristic dimensions w and l_a, of the active zone (Fig. 1). Those results are only qualitative because damage distribution within the zone is not homogeneous.

A complete experimental characterization of the kinematics of a damage zone requires comparison of damage distribution within the zone at different configurations. Moreover, the process of a CL growth is, in fact, a stochastic process. Thus the distributions of a sufficiently large number of identical systems should be compared at each configuration. Such an effort, however, is experimentally difficult to carry out. Independent studies have shown that in PS the macroscopic behaviour of a CL is well reproducible. In addition, analysis of craze distribution along the trailing edge of the active zone (Fig. 1) in specimens fatigued under the same loading conditions has shown that, at identical crack lengths, the distributions are the same [27]. These data are sufficient indication that craze distribution within an active zone is reproducible.

Botsis [28] has shown that during rectilinear CL propagation in PS, crazing is uniformly distributed in the thickness direction. Therefore, sections parallel to the plane of the specimen adequately represent craze distribution within a CL zone. In addition, no changes in orientation of crazes are observed during CL propagation. Accordingly, we characterize damage as the area of craze middle plane per unit test volume, ρ(mm^2 mm^{-3}).

For our measurements. sections parallel to the

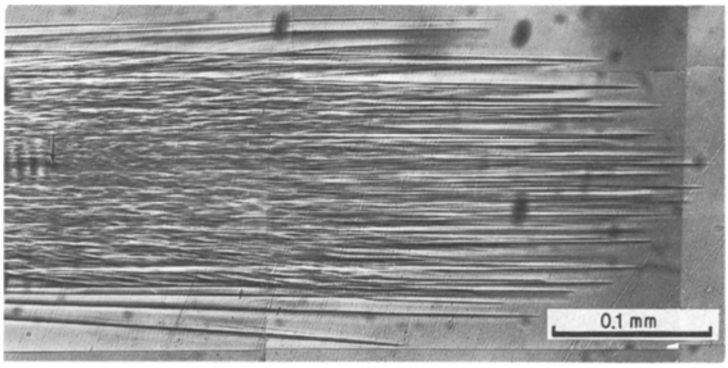

Figure 2 Optical micrograph of an active zone after thinning. Arrow points to the crack tip.

plane of the specimens were obtained by standard metallographic and polishing techniques [25]. A typical micrograph of a thinned section is shown in Fig. 2. From micrographs similar to that shown in Fig. 2, distributions of damage density within the active zone are obtained following the procedures which are described by Botsis [28].

Measurements within the active zone at six locations (Section 2) yield the contours of equal level of crazing density, ρ, shown in Fig. 3 (polygonal lines). Using non-linear regression analysis [29], the contours are approximated by ellipses (continuous lines, Fig. 3). Note that for the sake of visual clarity, the level of experimental error is not shown in Fig. 3. The error in the craze density on contours with density, $\rho = 300$, 600, 900, and $1200\,\mathrm{mm}^2\,\mathrm{mm}^{-3}$ is about 15%. The error on the contours with the highest craze density is about 30%. This is because the high craze density in the close vicinity of the crack tip limits optical microscopy which makes accurate measurements difficult. From the experimental data presented in Fig. 3 the weighted centre of gravity of the active zone with reference to a coordinate system attached at the crack tip is calculated. The results of these calculations are demonstrated in Fig. 4. These data indicate that the centre of gravity of the active zone x, increases with crack length.

Fig. 5 shows the evolution of the crack growth kinetics, $\dot{l}$, and the speed of the active zone centre, $\dot{L} = \dot{l} + \dot{x}$, as a function of the crack length. Note that both $\dot{l}$ and $\dot{L}$ practically coincide because the value of x is small compared to the respective crack length.

One way to investigate the kinematics of the active zone is to examine the contours of equal damage

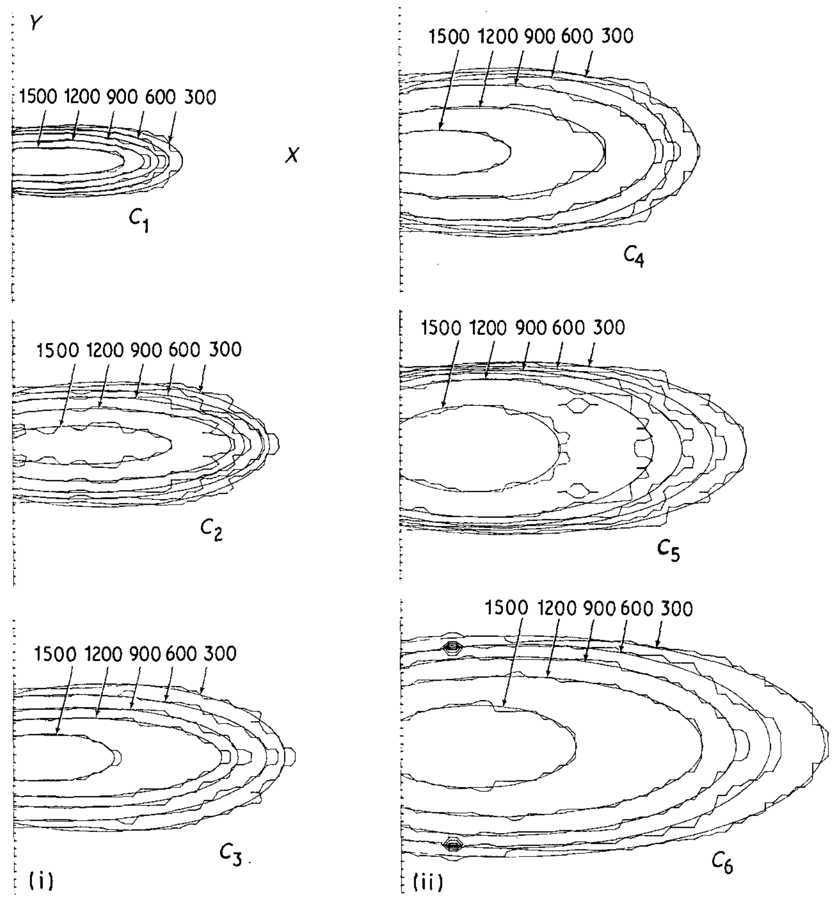

Figure 3 Contours of equal damage density (polygonal lines) $\rho(\mathrm{mm}^2\,\mathrm{mm}^{-3})$, and their elliptical approximations (continuous lines). C_i indicates configurations.

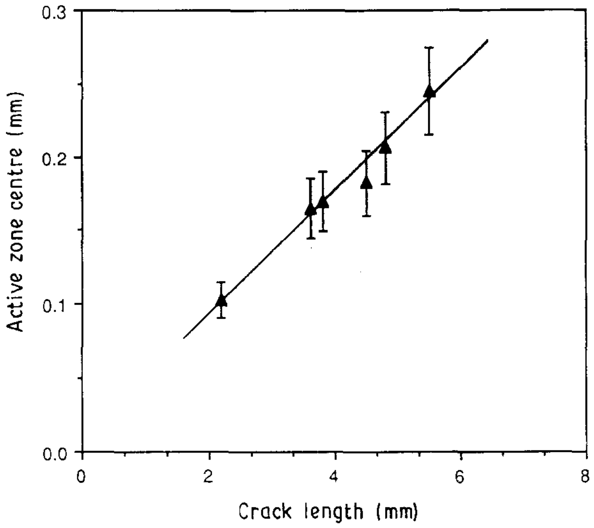

Figure 4 Evolution of the weighted centre of gravity of the active zone as a function of the crack length.

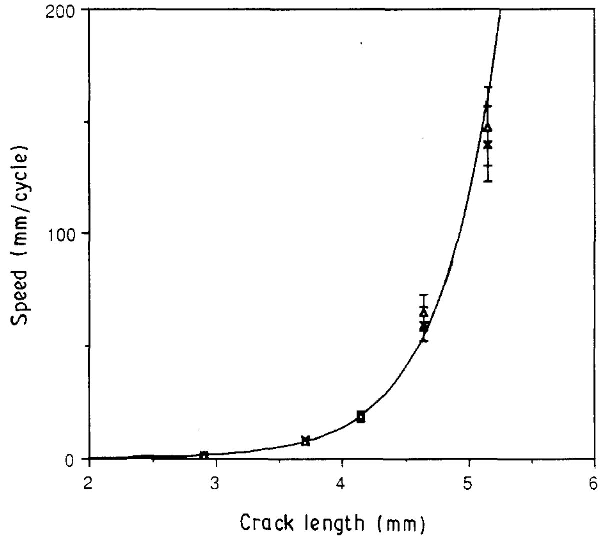

Figure 5 The speeds of ($\times$) crack, $\dot{l}$ and (Δ) the weighted centre of the active zone, $\dot{L}$, plotted against the crack length.

density at consecutive configurations. Such pointwise comparison however, may not be appropriate because of built-in uncertainty. Indeed, the contours of equal damage level at each configuration obtained from different specimens exhibit noticeable fluctuation on the scale of several micrometers (Fig. 3). Therefore, integral parameters are introduced to characterize damage distribution and its evolution. These are the centre of gravity, the moments of inertia of the active zone and total amount of crazes within the zone.

For the purpose of investigating what type of transformation the damage density is undergoing during fracture, we utilize the moments of inertia of the active zone. Through the moments of inertia characteristic scales along the X and Y axes can be defined,

$$\frac{I_{2x}}{I_0} = \sigma_x^2, \qquad \frac{I_{2y}}{I_0} = \sigma_y^2$$

Here I_{2x}, I_{2y} are the second moments of inertia of the active zone with reference to a coordinate system attached to the centre of gravity of the zone and I_0 is the zeroth moment which expresses the total amount of crazes within the zone. Thus, σ_x and σ_y can be looked upon as measures of the spread of damage in the X and Y axes, respectively (Fig. 3).

On the basis of these parameters we examine damage evolution within the active zone. For this purpose, the following ratios are defined between ith and jth configurations ($i = 1, \ldots 5$ and $i < j \leq 6$)

$$\frac{\sigma_x^{(j)}}{\sigma_x^{(i)}} = \lambda_{ji}, \qquad \frac{\sigma_y^{(j)}}{\sigma_y^{(i)}} = \mu_{ji}$$

Below we consider only consecutive configurations, i.e. $j = i + 1$.

Examination of damage evolution should involve comparison of a sufficiently large number of inertia moments of damage distribution between consecutive configurations. In this analysis we limit the comparison to the second and fourth moments only. Accordingly, we define the following ratios,

$$\frac{I_{4x}^{(j)}/I_0^{(j)}}{I_{4x}^{(i)}/I_0^{(i)}} = \Lambda_{ji}^4, \qquad \frac{I_{4y}^{(j)}/I_0^{(j)}}{I_{4y}^{(i)}/I_0^{(i)}} = M_{ji}^4, \qquad \frac{I_{2x\,2y}^{(j)}/I_0^{(j)}}{I_{2x\,2y}^{(i)}/I_0^{(i)}} = H_{ji}^4$$

where $I_{4x}^{(j)}$, $I_{4y}^{(j)}$ and $I_{2x\,2y}^{(j)}$ are the fourth rank moments of inertia of the active zone, at configuration j, with respect to a coordinate system attached to the centre of gravity of the zone and $I_0^{(j)}$ is the total amount of crazes within the zone at the respective configuration.

The type of transformation of damage between consecutive configurations is examined by comparing the following ratios; $\lambda_{ji}^4/\Lambda_{ji}^4$, μ_{ji}^4/M_{ji}^4, and $\lambda_{ji}^2\mu_{ji}^2/H_{ji}^4$. The data in Fig. 6 demonstrate that $\lambda_{ji}/\Lambda_{ji}$, μ_{ji}/M_{ji} and $\lambda_{ji}^2\mu_{ji}^2/H_{ji}^4 \approx 1$. These equalities indicate that the evolution of damage between consecutive configurations can be approximated by a linear transformation of the space variables.

Figure 7 demonstrates the evolution of $\sigma_x^{(j)}/\sigma_x^{(1)}$ (Fig. 7a) and $\sigma_y^{(j)}/\sigma_y^{(1)}$ (Fig. 7b) ($j = 1, \ldots 6$). $\sigma_x^{(1)}$, $\sigma_y^{(1)}$ refer to the spread of damage in the X and Y axes at configuration 1. The increase in the values of these ratios with the crack length shows that crack growth is accompanied by both translation and deformation of the active zone with the corresponding energy release

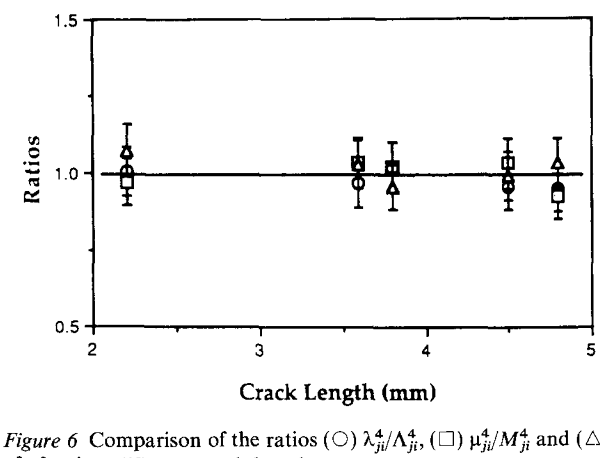

Figure 6 Comparison of the ratios ($\bigcirc$) $\lambda_{ji}^4/\Lambda_{ji}^4$, ($\square$) μ_{ji}^4/M_{ji}^4 and ($\triangle$) $\lambda_{ji}^2\mu_{ji}^2/H_{ji}^4$ at different crack lengths.

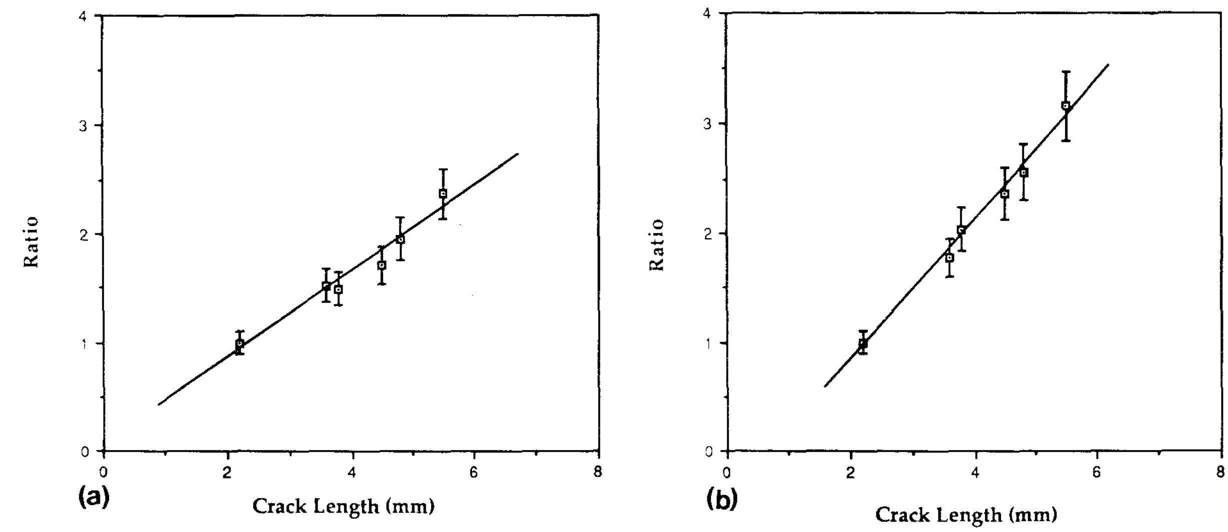

Figure 7 Evolution of the ratios (a) $\sigma_x^{(i)}/\sigma_x^{(1)}$ and (b) $\sigma_y^{(i)}/\sigma_y^{(1)}$ with crack length.

rates being given by the J_1, M, and N_{ij} integrals (rotation of the zone is not observed; this is expected because of symmetry in specimen geometry and applied load) [20].

4. Conclusion

Analysis of the kinematics of damage during quasi-static fracture in a model material shows that a finite number of parameters could be employed to approximate the kinematics of a damage field. The experimental results indicate that a linear transformation of points with equal damage density is an adequate approximation for the kinematics of the particular craze zone. The results of the preceding analysis support the particular form of self-similarity hypothesis of damage evolution proposed by Chudnovsky [19, 20] and is in agreement with the kinematics reported by Chudnovsky *et al.* [21] and Aoki *et al.* [23].

For the particular loading conditions the analysis shows that fracture propagates by translation and deformation of the active zone. Correspondingly, the energy release rates are given by the J_1, M and N_{ij} integrals.

Damage dissemination is a material- and loading-history-dependent process. Therefore in order to make general statements regarding the kinematics of damage during fracture propagation, experimental data under a wide spectrum of loading conditions and for a variety of material structures should be available.

Acknowledgements

The authors acknowledge financial support from the AFOSR under grant 89-0505. Thanks are also due to B. Kunin for useful discussions.

References

1. K. HELLAN, "Introduction to Fracture Mechanics" (McGraw-Hill, New York, 1984).
2. H. E. ANDREWS and J. B. WALKER, *Proc. Roy. Soc.* **A325** (1971) 57.
3. E. P. BRETZ, R. W. BRETZ, R. W. HERTZDERG and J. A. MANSON, *Polymer* **22** (1981) 1272.
4. A. CHUDNOVSKY, A. MOET, R. J. BANKERT and T. M. TAKEMORI, *J. Appl. Phys.* **54** (1985) 5562.
5. A. SANDT and E. HORNBOGEN, *J. Mater. Sci.* **19** (1981) 2915.
6. J. BOTSIS, A. CHUDNOVSKY and A. MOET, *Int. J. Fract.* **33** (1987) 263.
7. R. H. HOAGLAND, T. G. HAHN and A. R. ROSENFIELD, *Rock Mech.* **5** (1973) 77.
8. M. T. TAKEMORI and R. P. KAMBOUR, *J. Mater. Sci.* **16** (1981) 1110.
9. N. J. MILLS and J. N. WALKER, *ibid.* **15** (1980) 1840.
10. W. E. ANDREWS and S. P. BARNES, in "International Conference on Deformation Yield and Fracture of Polymers" (The Plastics and Rubber Institute, Cambridge, UK, 1982) p. 8.1.
11. N. HADDAOUI, A. CHUDNOVSKY and A. MOET, *Polymer* **27** (1985) 1377.
12. G. A. EVANS and E. A. HEUER, *J. Amer. Ceram. Soc.* **63** (1980) 246.
13. A. CHUDNOVSKY and M. BESSENDORF, "Crack Layer Morphology and Toughness Characterization in Steels", NASA Report 168 154 (1983).
14. R. N. LANG, T. A. MANSON and R. W. HERTZBERG, *ACS Organ. Coating Appl. Polym. Sci.* **49** (1983) 48.
15. P. X. NGUYEN and A. MOET, *J. Vinyl Tech.* **7** (1985) 140.
16. *Idem, Polym. Composites* **8** (1987) 298.
17. E. F. BURECH, "Fracture", Vol. 3 (Pergamon, London, 1972) p. 929.
18. J. N. CLAUSSEN, *J. Amer. Ceram. Soc.* **59** (1976) 49.
19. A. CHUDNOVSKY, "Crack Layer Theory", NASA Report 174 636 (1984).
20. *Idem*, "Crack Layer Theory", in 10th US National Conference on Applied Mechanics, edited by J. P. Lamb (ASME, Austin, Texas, 1986) p. 97.
21. A. CHUDNOVSKY, V. DUNAEVSKY and V. A. KHANDOGI, *Arch. Mech.* **30** (1978) 165.
22. A. CHUDNOVSKY and B. GOMMERSTADT, *Int. J. Solids Struct.* **22** (1986) 721.
23. S. AOKI, K. KISHIMOTO and M. SAKATA, *J. Appl. Mech.* **48** (1981) 825.
24. V. A. KHANDOGI and A. CHUDNOVSKY, in "Dynamics and Strength of Aircraft Structures" (edited by K. M. Kurshin, in Russian) (Novosirbisk, 1978) p. 148.
25. S. A. HOLIK, R. P. KAMBOUR, D. G. FINK and S. Y. HOBBS, in "Microstructural Science", Vol. 7, edited by

LeMay, Fallon and McCall (Elsevier, North Holand, 1978) p. 357.
26. J. BOTSIS, *Polymer* **29** (1988) 457.
27. X. Q. ZHANG and J. BOTSIS, to be published.
28. J. BOTSIS, *J. Mater. Sci.* **24** (1989) 2018.
29. P. R. BEVINGTON, "Data Reduction and Error Analysis for the Physical Sciences" (McGraw-Hill, New York, 1969).

Received 5 December 1989
and accepted 19 March 1990